MOMENTS OF ORDER STATISTICS FROM DOUBLY TRUNCATED BURR XII DISTRIBUTION: A COMPLEMENTARY NOTE WITH APPLICATIONS

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Abstract

By using some of its distributional properties, we obtain some results on recurrence relations for single and product moments of order statistics from doubly truncated Burr XII distribution. These results complement earlier results of Begum and Parvin [2002], as well as, generalize results obtained by Balakrishnan et al. [1998] and Saran and Pushkarna [1999]. Simulation results for the mean of the r-th order statistics and for product moments of order statistics are obtained and are consistent with those obtained by Begum and Parvin [2002].

1. The Doubly Truncated Burr XII Distribution

The distribution function of a doubly truncated Burr XII distribution is given, Begum and Parvin [2002], by

$$F(x) = \begin{cases} 0; x < Q_1 \\ \dfrac{1 - Q - (1 + \theta x^{\alpha})^{-\lambda}}{P - Q}; Q_1 \le x \le P_1, \lambda, \theta, \alpha > 0 \\ 1; x > P_1 \end{cases} \tag{1.1}$$

and probability density function (pdf)

$$f(x) = \frac{\lambda \theta \alpha x^{\alpha - 1}(1 + \theta x^{\alpha})^{-(\lambda + 1)}}{P - Q}; Q_1 \le x \le P_1, \lambda, \theta, \alpha > 0 \tag{1.2}$$

where Q and (1-P), (Q<P) are the proportions of truncation on the left and right of the distribution respectively

$$Q_1 = \left[\frac{(1-Q)^{-\frac{1}{\lambda}} - 1}{\theta}\right]^{\frac{1}{\alpha}} \quad \text{and} \quad P_1 = \left[\frac{(1-P)^{\frac{-1}{\lambda}} - 1}{\theta}\right]^{\frac{1}{\alpha}}$$

The quantities Q and P are assumed to be known. Denote $Q_2 = \frac{1-Q}{P-Q}$ and $P_1 = \frac{1-P}{P-Q}$ it is easy to see that the pdf (1.2) satisfies the differential equation, (Begum and Parvin [2002]),

$$(1+\theta x^{\alpha}) f = \lambda\theta\alpha x^{\alpha}[P_2 + (1-F)] \qquad (1.3)$$

or equivalently

$$(1+\vartheta x^{\alpha}) f = \lambda\theta\alpha x^{\alpha-1}[Q_2 - F]. \qquad (1.4)$$

Let $X_{1:n} \le X_{2:n} \le .... \le X_{n:n}$ be order statistics from a continuous distribution function (df) F(x) and probability density function (pdf) f(x). Let

$$\mu_{r:n}{}^{(i)} = E[X_{r:n}{}^{i}], \quad 1 \le r \le n$$

and

$$\mu_{r:n}{}^{(i,j)} = E[X_{r:n}{}^{i} X_{s:n}{}^{j}], \quad 1 \le r < s \le n.$$

David [1970] gives the density function of $X_{r:n}$ $(1 \le r \le n)$ as

$$f_{r:n}(x) = C_{r:n}[F(x)]^{r-1}[1-F(x)]^{n-r} f(x) \qquad -\infty < x < \infty. \qquad (1.5)$$

where $C_{r:n} = \frac{n!}{(r-1)!(n-r)!}$ and the joint density function of x=$X_{r:n}$ and y=$X_{s:n}$ as

$$f_{r,s:n}(x,y) = C_{r,s:n}[F(x)]^{r-1}[F(y)-F(x)]^{s-r-1}[1-F(y)]^{n-s} \quad -\infty < x < y < \infty \qquad (1.6)$$

where $C_{r,s:n} = \frac{n!}{(r-1)!(s-r-1)!(n-s)!}$

By using the relations (1.3) and (1.4), we derive recurrence relations for the distribution (1.2). These [i]relations are then applied to least squares estimation of location-scale parameter. We consider examples involving singly and doubly censored life-testing data and some agricultural data in the presence of missing observations. We present numerical results for the vectors of the coefficients of the BLUE's of the location-scale parameter

(i) For non-truncated i.e, $Q=0$ and $P=1$, Balakrishnan et al. [1998] had considered Lomax distribution which follows as a special case for the recurrence relations of Burr XII distribution when $\alpha = 1$, $\theta = 1$ and $\lambda = \alpha$.

(ii) By using the relations (1.3) and (1.4) $\alpha = 0,\ \theta = 1$ and $\lambda = \alpha,$ Saran and Pushkarna [1999] have established the recurrence relations for the single and product moments of order statistics from doubly truncated Lomax distribution.

Tables 1 and 2 give the mean of single and product moments of order statistics from the doubly truncated Burr XII distribution for $\alpha = 1,\ \theta = 1,\ \lambda = 3,$ P = 0.95 and Q = 0.05.

Table 1: The Means of the r-th order statistics from doubly truncated Burr XII distribution
α=1, θ=1, λ=3, P=0.95 and Q=0.05

n/r	1	2	3	4	5	6	7	8	9	10
1	0.38444									
2	0.19621	0.57266								
3	0.13320	0.32222	0.69788							
4	0.10270	0.22478	0.41965	0.79062						
5	0.08482	0.17420	0.30059	0.49897	0.86354					
6	0.07311	0.14337	0.23582	0.36551	0.56570	0.92311				
7	0.06485	0.12266	0.19515	0.29005	0.42210	0.32313	0.97300			
8	0.05871	0.10779	0.16727	0.24161	0.33848	0.47228	0.67341	1.01591		
9	0.05398	0.09661	0.14697	0.20788	0.28378	0.38224	0.51731	0.71802	1.05315	
10	0.05021	0.08787	0.13152	0.18302	0.24517	0.32239	0.42214	0.55809	0.75800	1.08595

Table 2: Product Moments of order statistics from doubly truncated Burr XII distribution
α=1, θ=1, λ=3, P=0.95 and Q=0.05

n.	s./r	1	2	3	4	5
1	1	0.27993				
2	1	0.07525				
	2	0.14779	0.48464			
3	1	0.03309				
	2	0.05910	0.15957			
	3	0.10810	0.27618	0.64717		
4	1	0.01871				
	2	0.03172	0.07623			
	3	0.05207	0.12090	0.24290		
	4	0.08935	0.20164	0.39108	0.78192	
5	1	0.1220				
	2	0.02005	0.04470			
	3	0.03104	0.06741	0.12353		
	4	0.04800	0.10227	0.18368	0.32250	
	5	0.07827	0.16395	0.28903	0.4924	0.89678

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